

## Bottom Stress Measurements in Shallow Water under Waves

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**Abstract** In the constant stress part of the bottom boundary layer, from 0.1 to 2 m above the bottom, waves can bias the measurement of average correlation of streamwise and vertical velocity fluctuations due to tides and wind, the Reynolds stress. From four vector velocity sensors in 15 meters depth exposed to ocean swell and wind waves, the differencing technique of Trowbridge has been used to remove the wave bias. However determination of the wave boundary layer stress of the lowest centimeter, responsible for sediment erosion and suspension, remains elusive.

### Introduction

In steady flow in a uniform boundary layer, the stress is constant over a vertical interval of several meters where turbulent exchange of fluid carries momentum from the interior of the flow towards the boundary. In this exchange of fluid, scalar properties giving rise to density variations are also mixed. Within this actively mixing region, the stress can be measured by the correlation of velocity fluctuation in the streamwise direction with fluctuations of vertical velocity. Averaged over an interval long compared to the period of the turbulent eddies but still short enough for the flow to be considered stationary, this Reynolds stress, as the density scaled average correlation is called, is the boundary stress felt by the flow above. This stress in steady state can balance pressure gradients such as tidal forcing and surface applied wind stress. Since this stress is constant over the actively mixing boundary layer, it can be measured at any suitable height. However if the stress is measured very close to the bed, two problems arise. The first is that the boundary layer may no longer be uniform due to topography. Bottom roughness from grains or bed forms may accelerate or separate the flow over individual features and the stress will be laterally inhomogeneous. Farther from the bed, these variations will average out and the flow can be considered uniform. The second problem is the wave boundary layer, a thin region where the growth of a turbulent boundary layer in response to the horizontal near-bed wave velocities is not steady. In the wave boundary layer, wave velocity reversal occurs before the boundary layer thickens more than a few millimeters. For flow at a greater distance than this from the boundary, there is only an apparent roughness influence

from this wave boundary layer. The turbulent structure outside the wave boundary layer is that of a constant stress layer with bottom roughness enhanced over the physical roughness. Yet from the perspective of the sediment exposed to the wave boundary layer, it is the stress within the wave boundary layer that causes erosion and suspension.

### Measurements of Reynolds Stress

Outside the wave boundary layer, Reynolds stress can be computed from vector velocity measurements made with sufficient precision and accuracy. Four vector flow sensors were used in a deployment at 15 meters depth as part of the US Office of Naval Research's Coupled Boundary Layer Air-Sea Transport (CBLAST) experiment in Massachusetts at the Martha's Vineyard Coastal Observatory (MVCO). Two Modular Acoustic Velocity Sensors (MAVS) made by Nobska Development and two Acoustic Doppler Velocimeters (ADV) made by SonTek were deployed on a bottom frame with sensor volumes approximately 67 cm above the bed. The bottom frame, shown in Fig. 1, is a saw-horse structure without horizontal members on the bed that could distort a tidal flow. It has a horizontal bar at 1.25 meters to which the sensors are attached and is 7 meters long with sloped legs at the ends and in the middle. This bottom frame was aligned normal to the east-west tidal flow, three kilometers south of the south facing beach of Martha's Vineyard. The individual instruments were connected to a collector of serial data signals, a Mininode that was connected by Ethernet to a node of MVCO on a nearby tower from which it drew power. MVCO is connected by seafloor cable to shore on Martha's Vineyard from which it derives power and connectivity to Woods Hole Oceanographic Institution and the world by Internet.

Measurements of flow from all four sensors are triggered to start simultaneously by the PC104 computer in the Mininode and records of 21600 samples made at 20 Hz (18 minute bursts) from each sensor are stored every 20 minutes. These blocks of data are stored on hard drives in the Mininode for up to a month but are uploaded to a shore computer every hour. This scheme allows for failure of the island to mainland T1 line or other communications connections for extended periods

without loss of data. A backup generator in the shore lab where the cable terminates keeps data collection going during storms that interrupt commercial power distribution as well. In practice, communications are interrupted for other causes such as diver servicing of submarine connections to the node, but the system is resistant to normal storm interruptions.



Fig. 1. A MAVS current meter (left with rings) and an ADV current meter (right with fingers) are two of the four current sensors on this bottom frame. The sensing volumes are 66 cm and 68 cm respectively above the bottom and the pair wise separations of the sensors vary from 90 to 558 cm. The Mininode is visible between the legs on the left end of the frame. This frame is deployed in 15 meters depth three kilometers south of Martha's Vineyard, MA. The frame is aligned north and south so that the east-west tidal current passes through the open side. Waves generally come from the southwest, oblique to the open side.

### Reynolds Stress under Waves

At the CBLAST location, exposed to open Atlantic Ocean swell from SE through S to W, wave orbital velocities can often exceed tidal and wind set up currents. The Reynolds correlation technique is vulnerable to large wave velocities that can leak wave orbital velocities into apparent Reynolds stress correlations where none are expected. Slight uncorrected tilt of the sensor can convert zero mean horizontal wave velocity components into apparent correlations of horizontal with vertical fluctuations. Bottom topography can similarly cause stress-free wave orbits to appear to carry stress when measured by a properly plumbed sensor.

Two techniques are used to minimize this source of error. The first is spectral cut off for those cases where the wave energy is separated spectrally from the momentum-carrying turbulent eddies. The other is the technique of Trowbridge [1] where samples from horizontally separated sensors are differenced before

forming the correlations illustrated in Figs. 2-6 with MAVS data from the bottom frame.

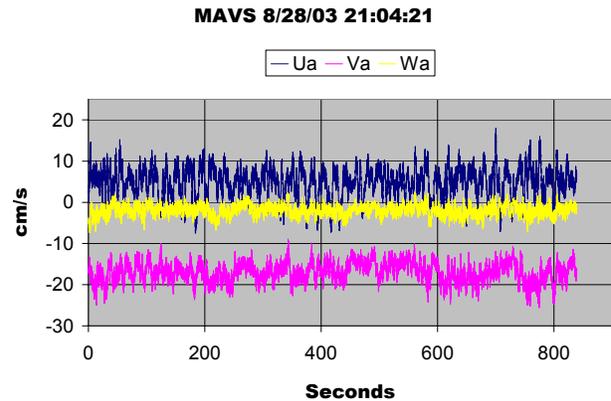


Fig. 2. The record from the MAVS A current meter shows a burst of 900 seconds at 20 Hz sample rate in MAVS coordinates. The V direction is approximately parallel to the shore showing a tidal current of 18 cm/s in a westerly direction.

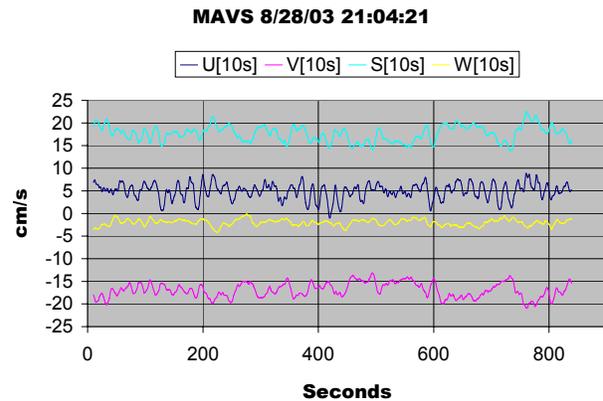


Fig. 3 Ten-second average of the velocity time series from MAVS A shown in Fig. 2. About 8 % of the tidal velocity shows up in the W signal, a possible 5° uncorrected tilt. Streamwise velocity vector, S is shown in cyan. This is the projection of the instantaneous U, V velocity vector onto the mean direction over the burst.

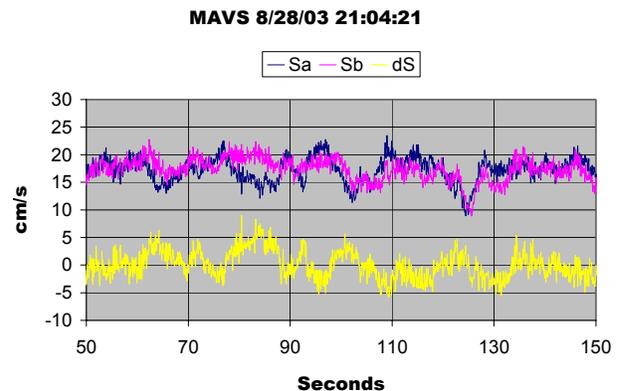


Fig. 4. Sa and Sb are the horizontal velocity components from MAVS A and MAVS B projected onto the streamwise direction. The difference, Sb-Sa, is plotted as dS. Only 100 seconds out of the 900 seconds analyzed are presented in this figure.

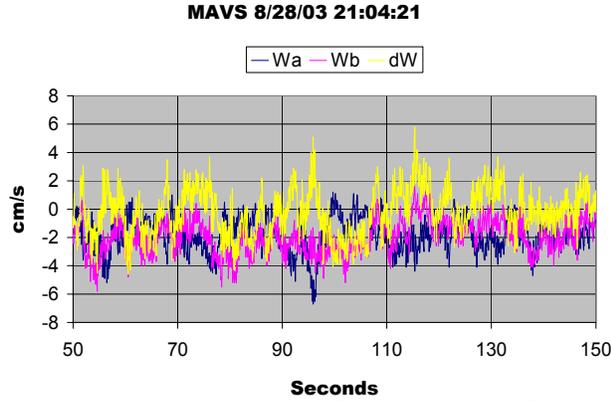


Fig. 5. The vertical component of velocity from MAVS A is subtracted from the vertical component of velocity from MAVS B. The difference is plotted as dW. Only 100 seconds of the 900 second record is shown. Note the leakage of V component of velocity into the W component due to uncorrected tilt.

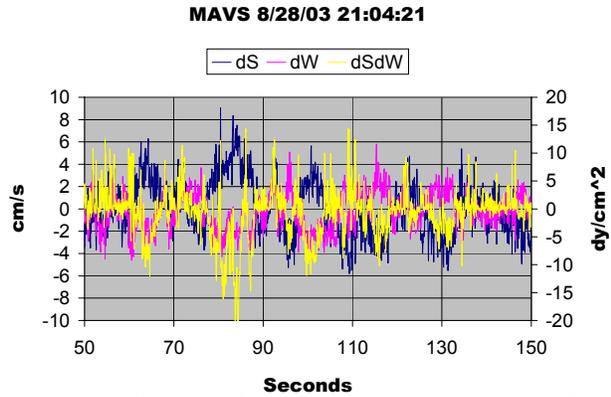


Fig. 6. Difference in fluctuations in streamwise velocity, dS, and in difference in fluctuations of vertical velocity, dW, are shown in blue and red respectively. The means of dS and dW are subtracted and the product indicated as dSdW in yellow. This is done for the entire burst but only 100 seconds are illustrated.

In the Trowbridge technique, wave velocities are correlated over the separations of the sensors while the turbulent eddies are not. After subtraction the wave components are attenuated. The turbulent eddies are essentially summed so the correlation used for Reynolds stress must be halved.

### Sweeps and Bursts

In a steady boundary layer flow, the eddies that are responsible for momentum transport across the boundary layer are termed sweeps when high velocity fluid moves towards the bottom and bursts when low velocity fluid is ejected into the flow more distant from the boundary. In each case the product of the fluctuations is negative. In Fig. 6 it can be seen that the excursions of the product are generally negative. But dS and dW are not fluctuations of velocity at a single sensor; they are the difference of velocity component fluctuations between two sensors. The appearance of a

burst or a sweep may reflect one sensor seeing it at a period when the other sensor is seeing little activity, but in general this technique is a statistical analysis that gives the correct result without detailed correspondence to specific eddy contributions.

### Trowbridge Method of Reynolds Decomposition

The Reynolds decomposition states that:

$$u = u' + U$$

$$w = w' + W$$

$\tau = -\rho \langle u'w' \rangle$ , where the lower case represents the instantaneous value, upper case represents average over the burst interval, the prime is the zero mean fluctuation, angle brackets indicate the average, and  $\tau$  and  $\rho$  are the Reynolds stress and the density of seawater (~1 gm/cc) respectively. U and W are the traditional formats but in this treatment, S is used to represent the streamwise velocity. Thus we will have:  $\tau = -\rho \langle s'w' \rangle$ , where  $s'$  is the fluctuation in the streamwise velocity.

These relations are true for both sensors so that:

$$\tau_a = -\rho \langle s_a'w_a' \rangle = \tau_b = -\rho \langle s_b'w_b' \rangle$$

where the subscript refers to the sensor that made the measurement.

Forming the differences and subtracting the means:

$$dS = s_b' - s_a'$$

$$dW = w_b' - w_a'$$

where the 'a' and 'b' quantities are uncorrelated because they are separated horizontally by more than the eddy correlation length.

Forming the product:

$$dSdW = (s_b' - s_a') * (w_b' - w_a')$$

$$= s_b'w_b' + s_a'w_a' - s_b'w_a' - s_a'w_b'$$

Taking the average, and multiplying by  $\rho$ , the equation becomes:

$$\rho dSdW = \langle s_b'w_b' \rangle + \langle s_a'w_a' \rangle - \langle s_b'w_a' \rangle - \langle s_a'w_b' \rangle$$

and since  $s_b'$  and  $w_a'$  are uncorrelated as are  $s_a'$  and  $w_b'$ ,  $\langle s_b'w_a' \rangle = 0$  and  $\langle s_a'w_b' \rangle = 0$ , leaving

$$\rho dSdW = -\tau_b - \tau_a = -2\tau$$

Thus by the method of Trowbridge:

$$\tau = -\frac{1}{2} \rho dSdW$$

### Wave Suppression by the Method of Trowbridge

Fig. 7 shows a period slightly later where the tidal current was reduced but the wave climate was elevated. Fig. 8 shows filtered velocity components from this interval. The 10 second means do not suppress the 20 second period swell that is present chiefly along the U

axis. The two observations of velocity are strongly correlated for waves. The small mean current allows a streamwise coordinate to be determined and this coordinate is  $S$ , used in Fig. 9.

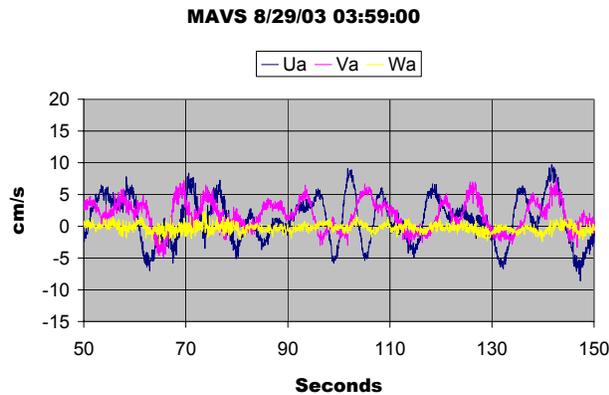


Fig. 7. MAVS A measurements where the current was weak but waves were present. Only 100 seconds are shown of the 1080 seconds of the burst.

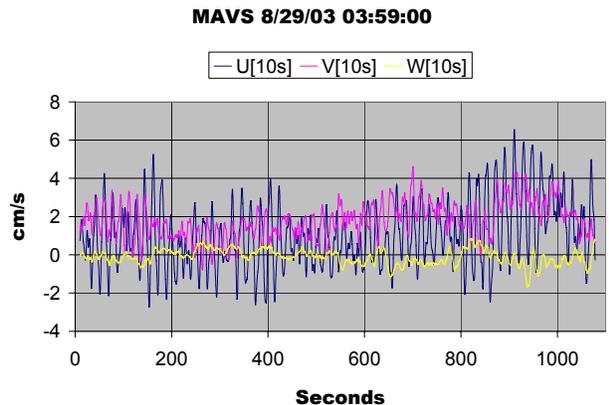


Fig. 8. This filtered MAVS velocity component record shows wave velocities on the bottom in 15 meters depth from approximately the U direction (southwest) with 20 second period. Mean current was less than 3 cm/s.

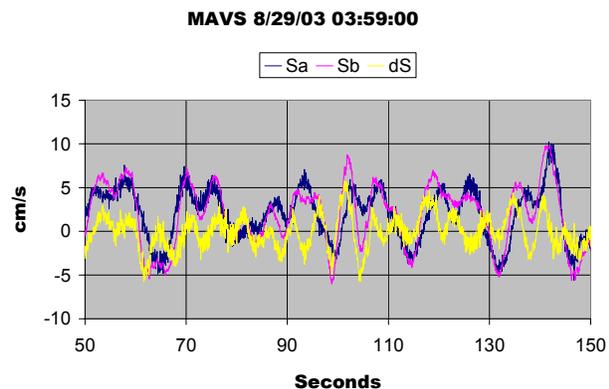


Fig. 9. The highly correlated wave velocities in the streamwise direction reduce to a small difference velocity,  $dS$ . This is a 100 second piece of the 1080 second burst.

The projection of the horizontal velocity components onto the streamwise direction and forming the

difference of these two projections reduces the wave amplitude substantially as shown in Fig. 9. Forming the Reynolds stress correlation shows little net stress. This is displayed in Fig. 10.

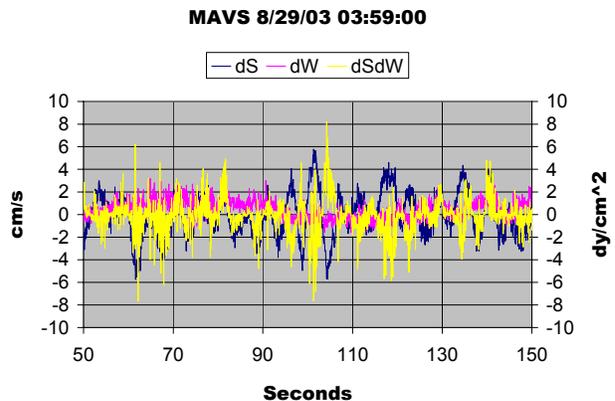


Fig. 10. The  $dSdW$  product in this low current but wavy record shows little net contribution. The negative excursions are balanced by positive excursions. This 100 second piece of the 1080 second burst shows individual turbulent eddies but they are not carrying momentum towards the bottom. In all probability they are wave boundary layer eddies that have diffused away from the boundary.

### Spectral Characteristics and Cutoffs

Fig. 11 shows the wave spectral peaks at 17 seconds and 5 seconds from MAVS sensor A, and the difference of the A and B sensors. The difference spectrum has removed the 5 second and attenuated the 17 second wave but there is an augmented feature at 3 Hz that was present in each of the single sensors. This appears to be a Strouhal oscillation from the 1.2 cm central strut of MAVS in the 17 cm/s current. It does not vanish because the wakes are incoherent.

When the mean velocity is less than the peak wave velocities, there is inevitably leakage of the wave energy into the covariance and spectral methods are revealing. Fig. 12 shows this condition.

After the winter fouling was removed from one MAVS current meter and the second was recovered entirely, data revealed that Reynolds stress could be derived in certain conditions without differencing. Fig. 13 shows this wave spectrum, while Fig. 14 shows the  $dSdW$  product responsible for the Reynolds stress.

### Summary

The statistics of these three bursts are summarized in Table I. The ADV measurements were calculated by a similar process to those of MAVS; however, while the MAVS covariance was simply averaged, the ADV covariance was integrated only to a cutoff that excluded the rising tail at high frequency and the 5.7 second waves at lower frequency. This difference allowed the

ADV to reject leakage that is present in the MAVS Reynolds stress. The MAVS determination of Reynolds stress as  $0.18 \text{ dy/cm}^2$  and consequently the drag coefficient,  $C_D$ , as  $0.0428$  under low-current high-wave conditions results in part from a remaining 5.7 second wave and 20 second swell. In addition, the separation of the ADVs was 275 cm in contrast to the 558 cm separation of the MAVS instruments, and because the ADVs have a higher electronic noise floor, the values from the two sets of calculations are somewhat different.

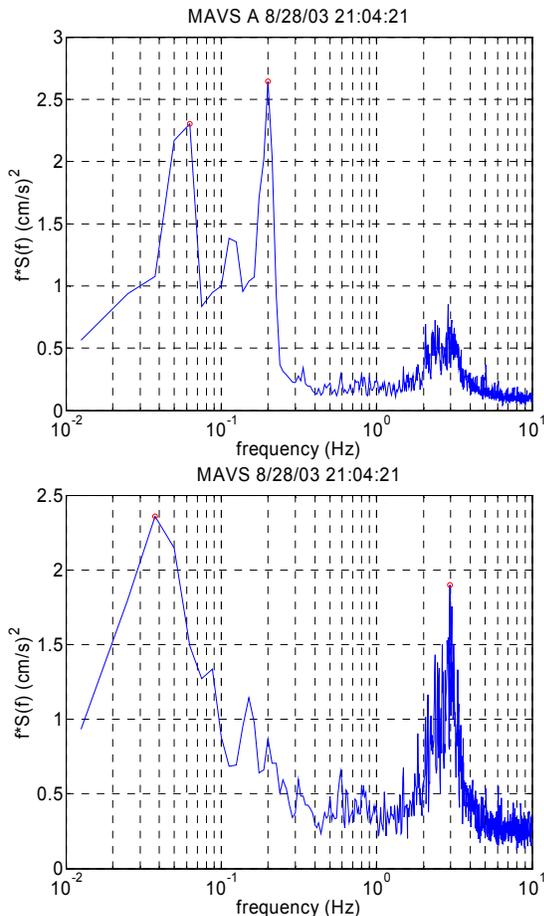


Fig. 11. The wave spectra from MAVS A and MAVS B are similar and when computed from the difference velocities of the two sensors, as shown in the bottom panel, the waves are largely gone. A spectral peak at 3 Hz is however summed implying it is similar in frequency but not in phase. This appears to be Strouhal oscillation from the 1.2 cm central strut of MAVS.

### MAVS Rake

Detailed simultaneous measurements of horizontal velocity can be made in the lowest five centimeters of the water column with an array of acoustic velocity sensors supported on vertical tines extending into the bottom. Spaced 3 mm apart vertically, acoustic paths can resolve the wave boundary layer velocities with

precision of 0.1 cm/s in two horizontal components. MAVS Rake has been designed but not yet built.

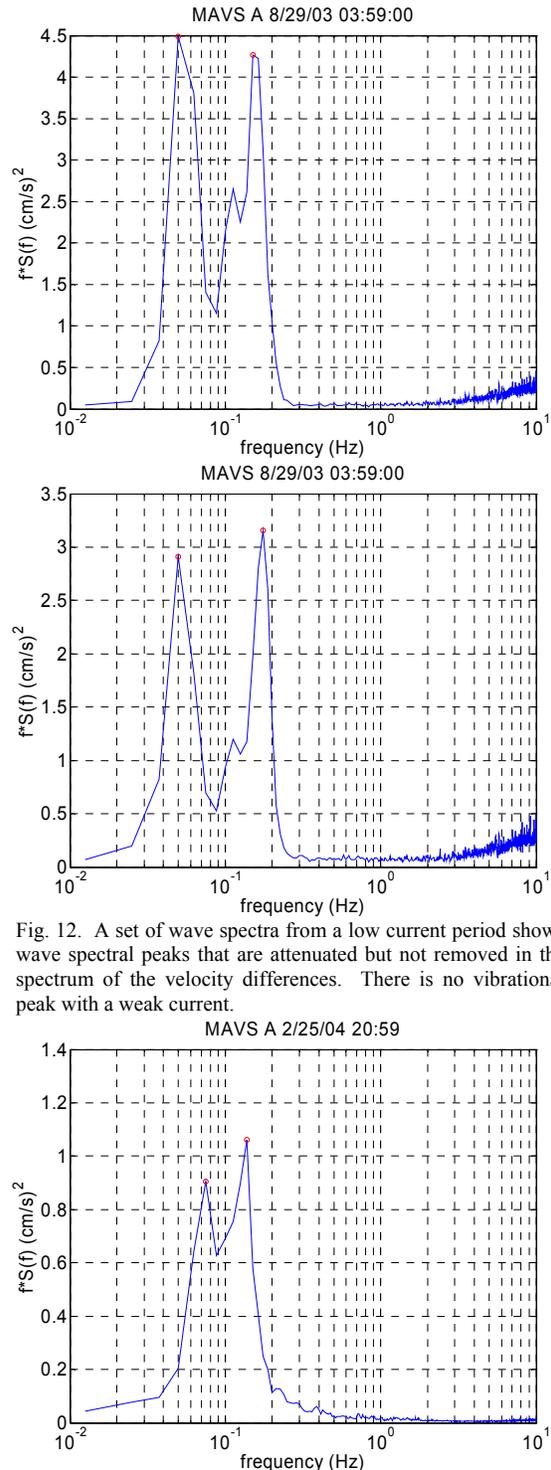


Fig. 12. A set of wave spectra from a low current period shows wave spectral peaks that are attenuated but not removed in the spectrum of the velocity differences. There is no vibrational peak with a weak current.

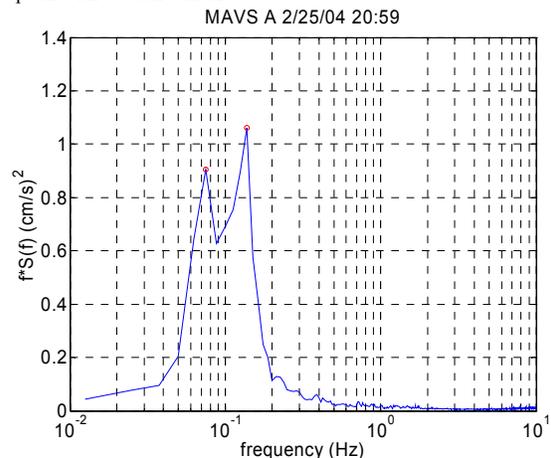


Fig. 13. Wave spectra from a single MAVS sensor after winter cleaning.

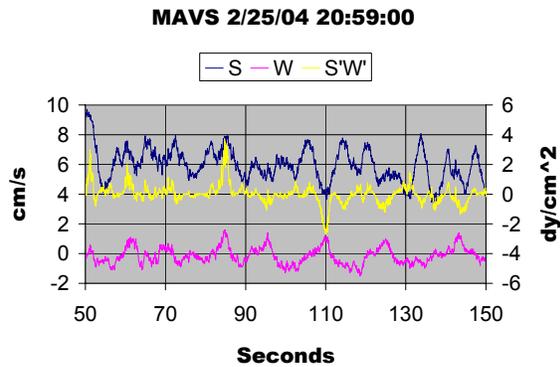


Fig. 14. The product of  $dS$  and  $dW$  can be averaged to give the Reynolds stress even though it is a single sensor in this case of non reversing wave flow.

Table I

Date – Time	8/28/03 – 21:04	8/28/03 – 21:40	8/29/03 – 03:59	8/29/03 – 03:59	2/25/04 – 20:59
Current meters	MAVS	ADV	MAVS	ADV	MAVS
Mean current	17.64 cm/s	12.9 cm/s	2.07 cm/s	2.09 cm/s	5.91 cm/s
Reynolds stress	0.61 dy/cm <sup>2</sup>	0.63 dy/cm <sup>2</sup>	0.18 dy/cm <sup>2</sup>	0.025 dy/cm <sup>2</sup>	0.092 dy/cm <sup>2</sup>
$u_*$	0.78 cm/s	0.79 cm/s*	0.43 cm/s	0.17 cm/s	0.30 cm/s
Wave period	5.7 s	5.7 s	5.3 & 17 s	5.3 & 17 s	13.3 s
$C_D$	0.0020	0.0038	0.0428	0.0057	0.0026

\*Note: Inertial dissipation gives  $u_* \sim 0.8$  cm/s,  $u_*^2 \sim 0.64$  cm<sup>2</sup>/s

A prototype based on a predecessor of MAVS, BASS, was deployed in the surf as a demonstration of the ability of such a vertical array of horizontal observations to capture the wave boundary layer velocity profile [2]. Fig. 15 is such a set of profiles over single wave periods showing velocity inversions as well as a shear magnitude near the bed. MAVS Rake is named for the current meter resolving the flow along the axes and the rake-like tines of the transducer supports. MAVS Rake was not deployed in the set of observations from the bottom frame at the MVCO so exact conditions at the sand bed cannot be known for sediment transport generating stress estimates. Had this sensor been included, peak velocity shear at the bed would have been used to characterize the erosion potential of the waves during each 18 minute burst of observations.

## Conclusion

The current is responsible for the Reynolds stress while the waves generate turbulence that does not contribute a

net correlation between fluctuations in the streamwise direction and the vertical. By the method of Trowbridge, the effect of waves can be reduced so that the Reynolds stress due to current can be measured. Spectral cut off in the covariance is required to remove residual waves and wake from structure of the sensor to achieve this result. This process was done with the ADV data but not with the MAVS data. There was no need to do this when the current dominated the wave velocity but was needed when the two velocities were similar.

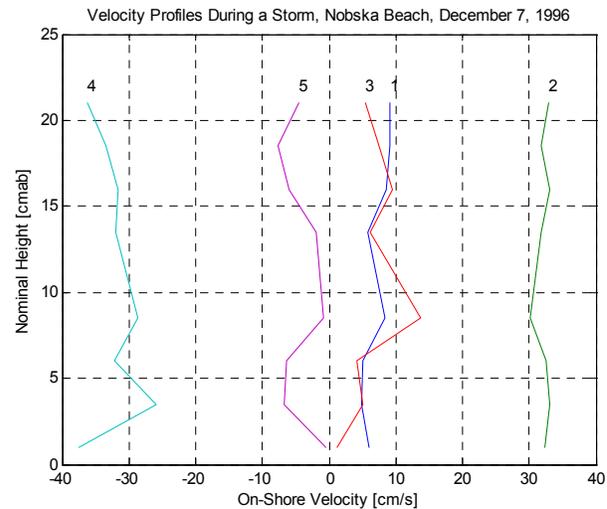


Fig. 15. BASS Rake profiles of velocity under storm waves during a single wave period. The velocity shear at the bottom of the profile reveals the stress that erodes and suspends sediment in the wave boundary layer. This scale is still somewhat greater than the actual wave boundary layer.

## Acknowledgement

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## Bibliography

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